Stress Constraints and Screening for Optimality Criteria Design

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Procedures that emphasize the processing of stress constraints within optimality criteria designs for low structural weight with stress and compliance constraints are described. Prescreening criteria are used to partition stress constraints into either potentially active primary sets or passive secondary sets that require minimal processing. Side-constraint boundaries for passive constraints are derived by projections from design histories to modify conventional stress-ratio boundaries. Other procedures described apply partial structural modification reanalysis to correct stress constraint violations of unfeasible designs. Sample problem results show effective design convergence and, in particular, advantages for reanalysis in obtaining lower feasible design weights.

Introduction

THERE have been many significant developments[1] during the past two decades in the structural-design optimization procedures that select member-size design variables to achieve low structure weight subject to constraints on displacement compliances and member stresses. For this type of problem, the difficulties of satisfying the stress constraints are usually more severe than those associated with the compliance constraints. Stress constraints are more troublesome because they are direct functions of strains; displacement constraints, however, are integral functions that benefit from the smoothing of local strain variations. This paper concentrates on the processing of stress constraints to expedite computations and to produce effective designs within optimality criteria methods. These are indirect design methods for which the solution space is equal to the number of active constraints and is typically smaller than the number of design variables. On the other hand, the solution space for direct design methods is equal in size to the number of design variables. Consequently, optimality criteria methods have the advantages of larger problem-size potential and smaller computational effort.

Optimality criteria solution methods determine Lagrange multipliers for the active constraints and then use these within the criteria to determine the design variables. The procedures to determine the Lagrange multipliers employ sensitivity (or flexibility) coefficients that are assumed not to depend upon the design variables. Nevertheless, the extent of structural redundancy weakens this assumption, and for the typically redundant structure it is necessary to develop the design through a number of iterative cycles. In each cycle: 1) the structure is reanalyzed to update the coefficients so that they are consistent with the current stage of design; 2) candidate active constraints are screened; 3) design space algorithms are applied to construct the active set of constraints and to determine the associated multipliers; and 4) the optimality criteria are applied to determine the design variables for the next cycle.

Nonlinear mathematical programming techniques are used to determine the Lagrange multipliers[2] for the set of active constraints. Methods for constructing this set apply hierarchical strategies in which constraints from a candidate set are assembled, added, or deleted. The determination of the multipliers and the construction of the sets of active constraints[3-5] are not covered here. Instead, the concern is with the screening procedures used to construct the candidate set of stress constraints from which the active constraints are subsequently selected. The objective is to reduce computations by making the candidate set small and yet retain the capability of achieving low-weight designs that satisfy the constraints.

The screening procedures discussed here consist of:

1) The well-known stress-ratio criterion, which rejects as primary constraints those stresses that are relatively small in comparison with the allowable stress.

2) A stress-resultant stability criterion that examines the change in stress resultants between adjacent design cycles and rejects as primary constraints those for which the relative change is small.

3) A second type of redundancy estimate criterion that is applied to individual components of vectors of auxiliary virtual work stress-resultant coefficients.

Stress constraints that do not pass all of the screening tests are converted to side constraints that specify minimum-size design variables. A modification proposed here for the traditional stress-ratio side constraint provides an adjustment that is based on a projected change in the stress resultant.

Constraint violations that result from structural redundancy will frequently occur during design. When this happens a hypothetically feasible design could be attained by scaling all design variables uniformly according to the worst relative constraint violation. A method of partial structural-modification reanalysis[6] described here provides an improved feasible design by developing adjustments for a set of controlling design variables. This permits a number of members to achieve their maximum allowable stresses simultaneously.

Procedures[7] for screening, side-constraint projection, and modification reanalysis restricted to single-member design-variable groups showed rapid convergence to low structure weight and short computation times for a number of test problems. These procedures are herein summarized and supplemented by new extensions for application to multiple-member design-variable groups. Results for sample design-problems using these supplemental procedures are provided.

Mathematical Formulations

A simplification in the following discussions is to consider three-dimensional truss-type structures that comprise only one-dimensional rod finite elements. The associated design variables are cross-sectional areas of rods, and the stress resultants are the axial forces. They can represent individual rod member areas or common areas for groups of members. The formulation, nevertheless, can be readily modified[8,9] to include additional membrane plate elements.

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Optimality Criteria Design

A condensation of typical formulations[2-5] is provided for reference and to establish the notation. The assumption of a common density for all structure elements permits using the structure volume V as an alternative objective to weight. The design objective then becomes

$$\text{Minimize } V=\sum_{i=1}^{N} L_i a_i \tag{1}$$

in which the design variable a_i is the cross-sectional area, L_i the length, and i the index within a set of N design variables. A set of K primary constraint equations is expressed as

$$G_j = C_j - C_j^* \leq 0 \qquad j=i,\ldots,K \tag{2}$$

where C_j^* is a specified upper limit on compliance and C_j the actual compliance given by

$$C_j=\sum_{i=1}^{N}\frac{F_{ij}L_i}{a_i} \tag{3}$$

In Eq. (3), F_{ij} is an internal stress-resultant, virtual-work (sensitivity) coefficient, so that the virtual work of the ith design variable for the jth constraint is $F_{ij}L_i/a_i$. When the design variable represents a single rod member, F_{ij} is computed as the stress resultant for the jth external loading times the stress resultant for an associated virtual "dummy" loading divided by Young's modulus. A simple illustration of the virtual loading[2] is when the displacement at a single node in a particular direction is to be constrained; then a unit load is applied at that node in the corresponding direction. In the case of a group of more than one individual member, F_{ij} is computed as the length-weighted average for the group.

The special case of a stress constraint can be converted to the following extensional constraint:

$$C_j^* = \sigma_r^* L_r / E \tag{4}$$

in which σ_r^* is the allowable stress and L_r the length of the rth member. The virtual loading is a pair of colinear self-equilibrating-unit virtual loads applied to the terminal nodes of the member, and E is Young's modulus. Other special[9] compliance constraints can be established from particular forms of virtual-loading vectors.

In addition to, or possibly in place of, the primary constraints, there are side constraints given by the limits

$$\underline{a}_i \leq a_i \leq \bar{a}_i \qquad i=1,\ldots,N \tag{5}$$

in which $\underline{a}_i$ and $\bar{a}_i$ are lower and upper side-constraint bounds on the design variable a_i.

Constraint ratios define the ratio of realized to allowable virtual work as

$$D_j = C_j / C_j^* \tag{6}$$

and values of D_j greater than unity indicate an unsatisfied constraint.

The familiar optimality criteria for this problem are:

$$a_i^2=\sum_{j=1}^{K} F_{ij}\lambda_j \qquad j=1,\ldots,K \tag{7}$$

in which the λ_j are non-negative Lagrange multipliers that are to be determined from the solution of an auxiliary problem given by

$$\lambda_j G_j = 0 \qquad j=1,\ldots,K \tag{8}$$

A set of indices Q can be associated with the set of currently active constraints, and the remaining K-Q indices can be associated with inactive constraints for which the multipliers are taken as zero. Then the auxiliary problem is to solve the nonlinear equations,

$$G_j = 0, \qquad j\epsilon Q \tag{9}$$

Stress Constraint Screening

The set of K primary constraints [Eq. (20)] is a candidate set from which the active set of Q constraints is established and associated Lagrange multipliers are derived to satisfy Eq. (9). The candidate set is developed by screening all of the possible stress and compliance constraints and discarding those that could produce unnecessary processing computations in the design procedures.

Three screening criteria[7] are described in the following paragraphs. The first two require only minimal computations. The third requires substantial computation because of the need for a self-equilibrating unit-virtual-load pair and computation of the associated stress-resultant vector for each application. Consequently, the third criterion is applied only to survivors of the first two. However, if a constraint survives the third criterion, this stress-resultant vector becomes a useful by-product because it is needed in the design to compute the virtual-work sensitivity coefficient. If no member of a design-variable group survives screening, a side constraint for the design-variable group is established according to the most severely stressed member of the group.

Screening criteria parameters that appear in subsequent discussions are listed with typical numerical values in Table 1.

Constraint Ratio Criterion

A constraint is rejected when the constraint ratio [Eq. (6)] is less than a specified threshold value t_c, typically moderately less than unity. This criterion can also be used for compliance constraints. The constraint ratio for stress constraints is equivalent to the stress ratio (actual/allowable). All individual members of a design-variable group are required to be rejected if the group is to be rejected.

Stress Resultant Stability Criterion

The relative change in stress resultant DP_i between adjacent design cycles for the ith member is computed as

$$DP_i = (S_i^+ - S_i^-)/S_i^+ \tag{10}$$

in which S_i^+ and S_i^- are the current and prior values of the stress resultant, respectively. Values close to zero indicate a stable stress resultant. If all members of a design-variable group have sufficiently low relative changes, it is reasonable to replace a primary stress constraint by a lower-bound design-variable side constraint.

The rejection criterion uses a threshold parameter t_s, which has its largest value at the initial design cycle and is progressively reduced by a multiplicative factor m_s less than unity. Furthermore, the relative change in stress resultant is augmented to reflect an overstress ratio. The criterion is:

$$DP_i + \max(D_i - 1.0, 0.0) < t_s \tag{11}$$

Table 1 Typical values or ranges for parameters

Application of parameter	Value or range
Constraint ratio screening criterion, t_c	0.95
Stress-resultant stability criterion, t_s	0.07
Stability criterion cyclic multiplier, m_s	0.65
Redundancy estimate criterion, t_r	0.90-0.95
Side-constraint projection, t_p	
For increasing stress resultant	0.15-0.25
For decreasing stress resultant	0.70-1.0
Cyclic multiplier, m_p	0.70-1.0
Modification reanalysis, t_m	0.99-1.01

Redundancy Estimate Criterion

The coefficient of the self-equilibrating, virtual-load pair, stress-resultant vector for a particular member at the index of that member is an indication of the influence of redundancy upon this member. If the absolute value is larger than a parameter t_r (close to unity), the member is considered to be "almost statically determinate." If all members of a member group exceed t_r, a primary stress constraint is replaced by a side constraint.[7,8]

Side Constraint Projection

Lower-bound side constraints are developed for all constraints that do not survive screening. The customary stress-ratio side constraint is

$$\underline{a}_i = a_i \sigma_i / \sigma_i^* \tag{12}$$

where σ_i is the current value of the stress for the most severely stressed member of the ith design-variable group. This side constraint can be modified slightly to anticipate redundancy using the equation

$$\underline{a}_i = S_i^* / \sigma_i^* \tag{13}$$

where S_i^* is the projected stress resultant given by

$$S_i^* = S_i^+ + t_p (S_i^+ - S_i^-) \tag{14}$$

in which t_p is an input parameter less than unity and the remaining terms are stress resultants as defined in conjunction with Eq. (10).

Figure 1a shows examples of stress-resultant cyclic histories for three bars during the execution of the 63-bar truss standard design problem.[13] These three bars were selected for illustration because they had primary stress constraints for almost all of the cycles and also because they exhibited relatively pronounced changes in stress resultants. As illustrated in Fig. 1b, changes in stress resultants proceed monotonically for long runs of cycles with very few reversals; also, the stress-resultant graphs approach linearity in the later cycle. Consequently, side constraints with stress-resultant projection could have been effective.

Figure 2 shows the errors in stress-resultant prediction made according to the usual assumption of invariance from the prior cycle and for projection according to Eq. (14). For the 39 projections shown, there are only 6 cases in which the projection method gives the poorer result. The worst relative performance of the projection method occurs for bar 52 at cycle 11 because, although there was a minor change in stress

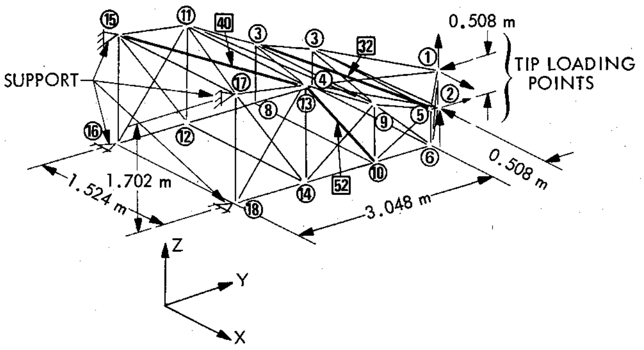

Fig. 1a Schematic of 63-bar truss.

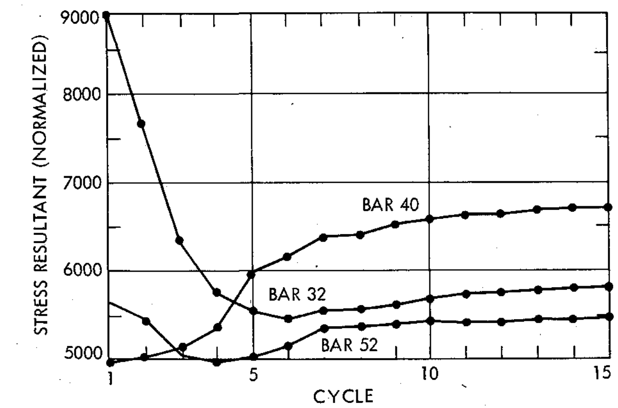

Fig. 1b Cyclic stress-resultant histories.

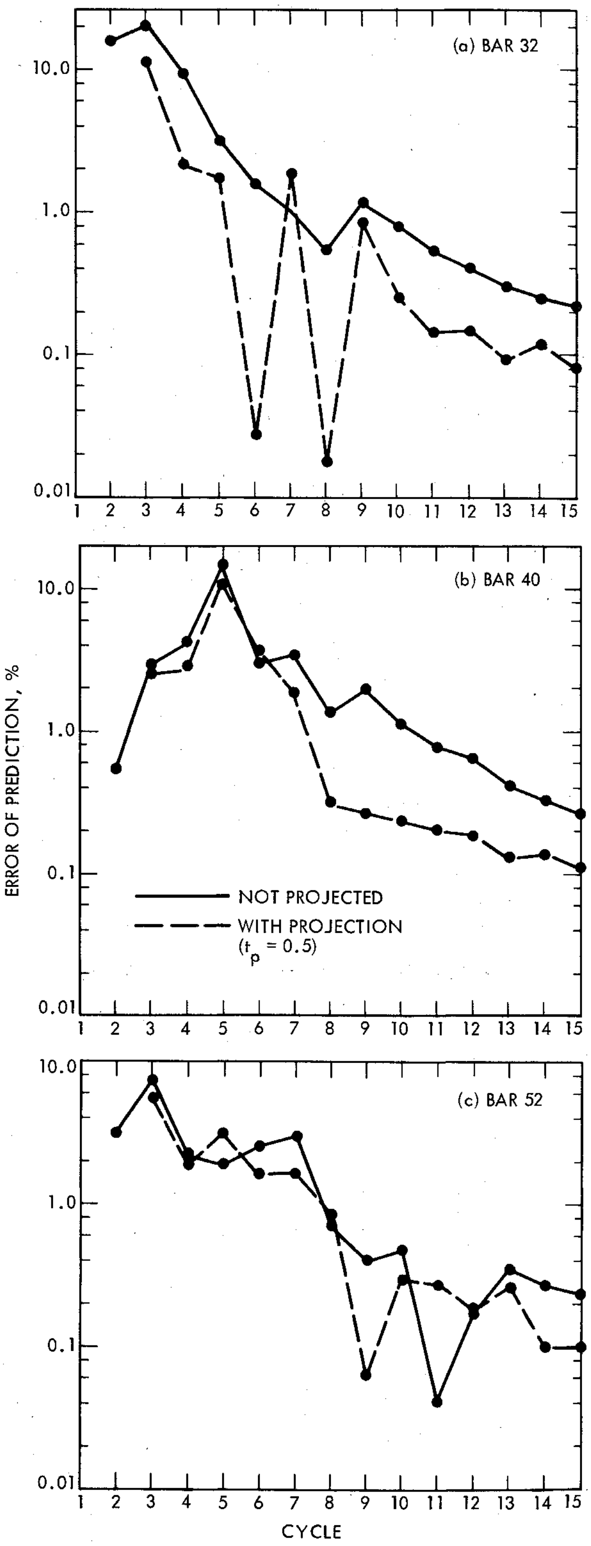

Fig. 2 Stress resultant prediction errors, 63-bar truss.

resultant from cycles 9 to 10, there was almost no change from cycle 10 to 11.

Structure Modification Reanalysis

Modification is used to adjust only the areas of the members with stresses greater than a threshold parameter t_m (Table 1). The purpose is to obtain a lower-weight feasible design than the design obtained by uniform scaling of all members.

The method derives from superposition of the external loading with linear combinations of auxiliary unit load pairs. Each pair consists of two loads placed at the end nodes of the member to be modified and directed toward each other. Therefore, except for possible differences in signs, the auxiliary loads are identical to the virtual loadings used to construct stress constraints for the same members.

To explain the method, assume that there are n_e members to be modified for external loading column e. Then let SS_e be the $n_e \times n_e$ vector of stress resultants for these members for the associated set of auxiliary loads, let R_e be a vector of magnitudes to be determined for the auxiliary loads, and let ΔS_e be a vector of changes in stress resultants. Then it follows from superposition that

$$\{\Delta S_e\} = [SS_e]\{R_e\} \tag{15}$$

For any one of the bars to be modified, say bar v, the required change in stress resultant ΔS_{ve} is equal to the allowable minus the current. That is,

$$\Delta S_{ve} = a_v \sigma_v^* - S_{ve} \qquad v = 1,\ldots,n_e \tag{16}$$

where a_v, σ_v^*, and S_{ve} are the area, allowable stress, and current stress resultant, respectively. After substituting these n_e definitions into the left-hand vector of Eq. (15), the equation is solved for R_e. The reanalysis concept used here[6] implies that the auxiliary loadings are applied to the structure to equilibrate the stress resultants of the hypothetical new bars placed in parallel with the originals. Consequently, R_{ve}, which is the vth component of the solution vector of Eq. (15), is the tensile load on a new bar with area Δa_v parallel to the original bar v. Since both the original and new bars have the same connectivity, they have the same stress. Therefore to satisfy the stress constraints, we require

$$\Delta a_v = R_{ve}/\sigma_v^* \tag{17}$$

which is the algorithm for the area change for member v for load column e. The procedure associated with Eqs. (15-17) is repeated for all other loading columns to determine the sets of area changes for the modified members. In case of conflicts for particular members, the selected change is the envelope maximum.

After the area changes are established, the stress resultants for all real and virtual loadings are updated to be consistent with the modifications. To do this, a new matrix R of multipliers for the auxiliary loads is computed and contains one column for each loading column to be updated. The requirement for this matrix is that the final stress resultants for the original bars minus the changes must be equal to the initial. This is expressed in an equation that is solved for R,

$$\Big[[\hat{a}/\Delta a] - [S_s]\Big][R] = [S_I S_s] \tag{18}$$

On the left side of this equation, S_s is the composite square matrix of stress resultants for the unit virtual-loading pairs for all of the modified members. The matrices on the right are assembled with row indices corresponding to those of the left, and the columns are the initial stress resultants for external loadings (S_I) and for the virtual loading vectors (S_s) associated with each constraint that survived the first two screening tests.

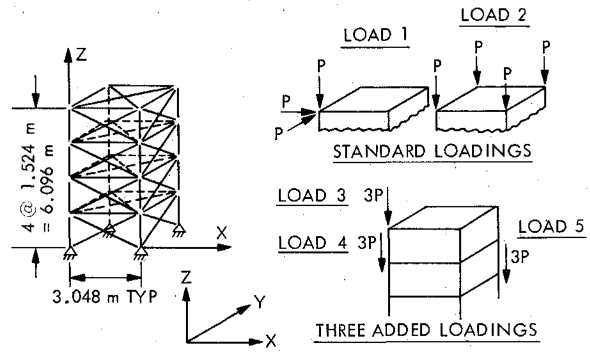

Fig. 3 72-bar truss.

When the matrix R has been obtained, a matrix of changes in stress resultants can be found from

$$[\Delta S] = \Big[[S_s] + [I_s]\Big][R] \tag{19}$$

in which the row indices of S_s are extended to account for all members, and I_s is a matrix with unity at the row and column indicies of modified members and is null elsewhere. An analogous expression that uses the same R matrix but omits the I_s matrix will also provide the matrix of changes in displacements for the modified structure. Reanalysis for the modified structure is completed by adding the changes to the original. To obtain the stress resultants, the calculation is

$$[S_I S_s]\text{modified} = [S_I S_s]\text{initial} + [\Delta S] \tag{20}$$

Except for the consequences of selecting area changes for more than one loading column by the envelope method, the stress ratios of the changed members in the modified structure will be unity. However, in the case of design-variable groups, an additional approximation occurs in the estimate of area change to change the member stress resultants. The approximation is necessary because it is not possible to specify independent changes in the stress resultants from Eq. (16) for individual members of a group since the area changes are constrained to be the same for all.

To develop the area changes, the partial derivative of the stress resultant S_{we} for bar w with respect to the area of bar v for external loading column e is given[7] by

$$\frac{\partial S_{we}}{\partial a_v} = \frac{S_{wv} S_{ve}}{\partial_v} \tag{21}$$

in which S_{wv} is the row w element of the stress-resultant vector for the pair of auxiliary loads associated with bar v, and S_{ve} is the stress resultant for bar v for column e. Then Eq. (21) provides the following approximation to the change in S_{we}:

$$\Delta S_{we} \simeq S_{wv} S_{ve} \Delta a_v / a_v \tag{22}$$

Equation (22) can be extended to furnish the vectors of all of the stress-resultant changes for this external-loading column that occurs for changes of the rth design variables as follows:

$$\{\Delta S\}_r = [SS]_r \{S_e\}_r (\Delta a/a)_r \tag{23}$$

In Eq. (23) the external subscript implies conformity with the rth group. That is, SS is the matrix of stress resultants for unit virtual-loading pair columns constructed for each member of this group, S_e is the vector of external-loading stress resultants for the members of the group, and Δa and a are the area change and the current area of the group.

Equation (23) can be recast as

$$\{\Delta S\}_r = [SS]_r \{f\}_r R_r \tag{24}$$

in which f is a vector derived by normalizing S_e to obtain a unit magnitude at the index for this group with the largest stress ratio. R_r is a scalar that, once found, will incorporate the normalization factor and the area terms. Furthermore, define the following vector:

$$\{SP\}_r = [SS]_r \{f\}_r \tag{25}$$

which, when inserted in Eq. (24), results in

$$\{\Delta S\}_r = \{SP\}_r R_r \tag{26}$$

Consequently, the approximation for the change in all the external-loading stress resultants for changes in p design-variable groups is obtained by incorporating all p right-hand sides constructed according to Eq. (26) in the single equation

$$\{\Delta S\} = [SP]\{R\} \tag{27}$$

The columns of the SP matrix in Eq. (27) consist of all right-side vectors developed from Eq. (26), and R is the vector that contains all the scalars. SP is then screened to extract only the rows corresponding to the member with the largest stress ratio of each group; these rows are assembled into a smaller ($p \times p$) coefficient matrix. Alternatively the screening could be initiated at Eq. (23) by restricting operations to the rows associated with the critical bar of each group.

Equation (16) is used to compute the required changes in stress resultants for the corresponding members and the condensed Eq. (27), which is analogous to Eq. (15), is solved for R. The remainder of the procedure, which employs Eqs. (17-20), follows as previously described.

Equation (20) is mathematically correct for any arbitrary set of area changes employed in Eq. (18). The approximations in these procedures relate to the way the area changes are derived for multiple-loading columns of multiple-member design-variable groups. The result of approximations is that the constraint ratios for the modified members may not be exactly unity after modification.

Examples

Standard test-design problems were processed by an earlier version of the present computer program that had limited capability to treat member-group design variables. The examples that follow are standard problems modified in an attempt to develop more substantive challenges for designs with group constraints. In the conventional definitions of some of these problems, it appears that either favorable characteristics of the external loading or regularity of the structure and member grouping permit the minimum structure weight to be attained with rapid convergence within a few design cycles.

Perhaps the best known of all design problems is that of the three-bar truss.[1-10] The structure is symmetrical and the problem can be formulated in terms of one external-loading case with two design-variable groups, one group consisting of the two sloping members and the other of the single central member. It requires only four design cycles to reach the minimum feasible weight and the weight at the second cycle is only 0.07% greater than the minimum. Structural modification reanalysis will provide the minimum weight at the second cycle but, otherwise, the problem presents no other significant features to the demonstration of the effective treatment of grouping constraints.

Another familiar and rapidly converging problem is the transmission tower with 25 bars.[11] The problem can be formulated[12] to consist of eight design-variable groups with two external loadings. The minimum feasible weights [247.3 kg (545.2 lbm)] can be attained in six cycles. However, the problem almost converges at the second cycle where the weight is only about 4% greater than the minimum.

72-Bar Truss

This structure is shown schematically in Fig. 3, and the specific details can be found in a number of references.[3,12] There are 16 design-variable groups, each of which contains 2-8 members of similar topology at each of the 4 levels. The standard problem specifies two external loading cases. The only binding constraints are the two displacement constraints in the X-Y plane at the point of application of the first loading, and the stresses for the four members of the uppermost post group for the second loading. At the third cycle, the convergence will be to within a trivial excess of the minimum weight. This rapid convergence may occur because the effective order of redundancy for a design with this grouping is much less than the order of redundancy for static analysis.[5]

In an effort to establish a less tractable problem with more significant stress constraints, the three additional loading cases shown in Fig. 3 were appended and the displacement constraint magnitudes were relaxed by a factor of two. In this design the stress constraints for the three uppermost post groups and the displacement constraints for the same components as in the standard problem became binding. Figure 4 shows the associated design history, which again indicates a benign problem. However, this figure illustrates that with

Table 2 Connectivity and design-variable grouping sets of 63-bar truss

	Node		Set			Node		Set			Node		Set			Node		Set	
Bar	1st	2nd	A	B	Bar	1st	2nd	A	B	Bar	1st	2nd	A	B	Bar	1st	2nd	A	B
1	1	3	1	1	17	3	5	9	9	33	6	8	19	17	49	7	4	35	23
2	2	4	2	2	18	4	6	9	9	34	7	13	20	18	50	9	14	36	24
3	1	5	1	1	19	7	9	10	10	35	8	14	21	19	51	7	12	37	24
4	2	6	2	2	20	8	10	10	10	36	9	11	22	18	52	13	10	38	24
5	7	3	3	3	21	11	13	11	11	37	10	12	23	19	53	11	8	39	24
6	8	4	4	4	22	12	14	11	11	38	11	17	24	20	54	13	18	40	25
7	9	5	3	3	23	1	2	12	12	39	12	18	25	21	55	11	16	41	25
8	10	6	4	4	24	3	4	13	13	40	13	15	26	20	56	17	14	42	25
9	11	7	5	5	25	5	6	13	13	41	14	16	27	21	57	15	12	43	25
10	12	8	6	6	26	7	8	14	14	42	1	6	28	22	58	5	4	44	26
11	13	9	5	5	27	9	10	14	14	43	1	4	29	22	59	3	6	45	26
12	14	10	6	6	28	11	12	15	15	44	5	2	30	22	60	9	8	46	26
13	15	11	7	7	29	13	14	15	15	45	3	2	31	22	61	7	10	47	26
14	16	12	8	8	30	3	9	16	16	46	5	10	32	23	62	13	12	48	26
15	17	13	7	7	31	4	10	17	17	47	3	8	33	23	63	11	14	49	26
16	18	14	8	8	32	5	7	18	16	48	9	6	34	23					

reanalysis the feasible weight at the second cycle is reduced from 10 to 2% greater than the least feasible weight. When reanalysis was permitted, it was used only at the second cycle because no stresses survived the screening parameters necessary to qualify for reanalysis at the third and fourth cycles.

63-Bar Truss

The conventional formulations of this design problem, which have 63 design variables in the absence of grouping constraints, are usually found to converge more slowly[4,5,13] than any of the previous examples. Therefore, modifications that added some group constraints were judged to be capable of providing a substantive problem to test the effectiveness of screening, side-constraint projection, and modification reanalysis. Two sets of grouping alternatives are defined in Table 2. Set A has 14 two-member and 35 single-member groups to provide a total of 49 design variables. Set B adds 6 additional two-member groups, 4 four-member groups, and 1 six-member groups to provide a total of 26 design variables that include only 1 single-member group.

Three alternative approaches were tested for each of three design problems:

1) Constraint screening and side-constraint projection were included but structure modification reanalysis was prevented.

2) The approach in alternative 1) was supplemented to permit modification reanalysis. For some of the problems, only selective modification was used. This prevented modification reanalysis for members with stress-constraint ratios either less than the maximum ratio of any of the members not selected for modification or less than the maximum displacement constraint ratio.

3) The same approach as alternative 2) was used, except that after screening and modification were completed all candidate primary stress constraints were eliminated and replaced by projected side constraints. Thus, only displacement constraints were considered during optimality criterion design.

Design histories for the three problems are shown in Fig. 5. The first two problems retain the standard stress and displacement constraints. The third problem has the same stress constraints but deletes all displacement constraints. Each of the 9 executions was made to continue through 15 cycles, which frequently exceeds the number needed for reasonable convergence. Relative computer times subsequently discussed are not precise because of the influence of different levels of voluminous printout requested for the computer runs. However, despite the anomalies due to printing, the slowest and fastest execution times differed by no more than 25% within each of the problems.

Figure 5a shows that reanalysis provided a significant reduction of the feasible weight achieved within the first few design cycles. Reanalysis with primary stress constraints

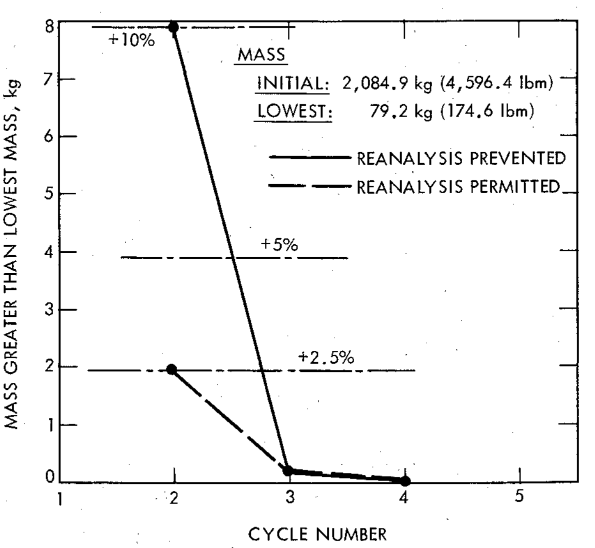

Fig. 4 Design history, 72-bar truss, five loads and relaxed displacement constraints.

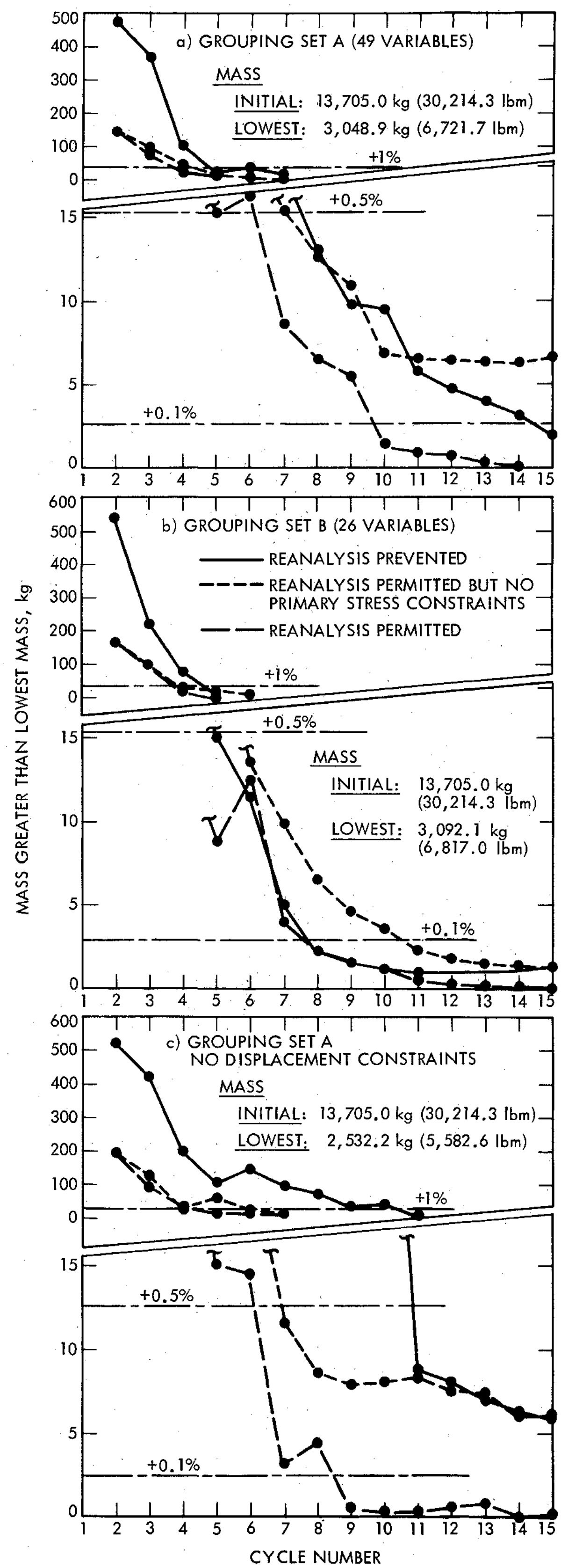

Fig. 5 Design history, 63-bar truss.

quickly attained low feasible weights that were only slightly higher than the minimum. When no primary stress constraints were permitted, the advantages of reanalysis occurred primarily within the first few cycles. Reanalysis with primary stress constraints took the most time for executing the 15 cycles; both of the other two approaches required about 20% less time.

Figure 5b illustrates a smoother and more rapid convergence to the lowest weight than is shown in Fig. 5a. This is attributed to the increased numbers of individual members contained within a smaller number of design-variable groups. The relative performance of the three strategies is similar to that described for Fig. 5a, except that, because the problem of 5b appears to be more tractable, the relative advantages for reanalysis are slightly less pronounced. It is interesting to note that the lowest weight for this problem is no more than 1.5% greater than for grouping Set A and the computation times are about one-third less.

For the problem of Fig. 5c, the displacement constraints are deleted to emphasize the influence of stress constraints. The omitted constraints had a significant effect which allowed these designs to be about 16% lighter. The usefulness of reanalysis is more apparent than in Figs. 5a and 5b because there is no stabilizing effect from displacement constraints. Even when no primary stress constraints are developed, there is a significant advantage in the feasible weight achieved with reanalysis through the first 10 cycles. Omission of the primary stress constraints for this problem invokes no optimization algorithm and requires the least computation time. The greatest amount of computation time is required for reanalysis with primary stress constraints, which is about 20% greater than the least computation time.

Conclusion

Structural design for practical structures will frequently entail analytical models comprising thousands of displacement degrees of freedom and members, and hundreds of design variables. These numbers are significantly larger than are those for typical problems considered within optimization reasearch, and the computer effort could become significantly greater than for conventional statics response analysis.

As a result, it is important to explore approaches that may facilitate the extension of research optimization to production-sized problems. A significant step, which has been substantively described elsewhere,[2,3] is the selection and effective implementation of optimality criterion as the design method.[4,5] Other opportunities to facilitate the applications to larger-sized problems that have been described here are:

1) Screening that eliminates potential primary stress constraints also eliminates the need for computations to determine sensitivity coefficients. These coefficients are costly to develop in practical designs because of the extensive use of auxiliary storage as well as the numerous operations required. Screening also can reduce the size of the solution space during the design procedure, which in turn can produce savings in a potentially extensive computational phase.

2) Side-constraint boundaries that improve upon the usual stress-ratio boundaries can provide better design convergence for the many more side constraints to be expected from screening.

3) Structural modification reanalysis can improve the convergence of feasible designs. This reanalysis also has the potential within the reduced numbers of the iteration cycles of achieving feasible designs with relatively low weight.

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